Convex Analysis and Nonlinear Optimization

Objectives: To describe various classes of convex optimization and some of their applications and extensions.
Prerequisites: Linear functional analysis, real and complex analysis, partial differential equations.

1. Background. Elements of Functional Analysis. Differential Calculus in Banach Spaces.
2. Inequality Constraints. Optimality Conditions. Theorems of the Alternative.
3. Fenchel Duality. Subgradients and Convex Functions. The Fenchel Conjugate.
4. Convex Analysis. Polar Calculus. Convex sets and Extreme Points. Fenchel Biconjugation. Lagrangian Duality. Duality for Linear and Semidefinite Programming. Convex Process duality.
5. Nonsmooth Optimization. Generalized Derivatives. Regularity and Strict Differentiability. Tangent Cones. The limiting Subdifferential. Karush-Kuhn-Tucker Theory.
6. Fixed Points. Variational Inequalities. Applications to PDE.

Bibliography

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3. F. H. Clarke, *Optimization and Nonsmooth Analysis*, Wiley, New York, 1983.
4. I. Ekeland and R. Temam, *Convex Analysis and Variational Problems*, Society for Industrial and Applied Mathematics, 1999.
5. J.-B. Hiriart-Urruty and C. Lemaréchal, *Fundamentals of convex aanalysis Algorithms*, Springer-Verlag, New York, 2001.
6. C.P. Niculescu and L.-E. Persson, *Convex Functions and their applications. A Contemporary Approach*. CMS Books in Mathematics, Springer-Verlag, New York, 2006.
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